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finds it "much easier to believe some of Campbell's identifications wrong than to believe that his measurements are incorrect by three and four tenth-metres." Campbell's measurements seem to the writer to constitute a proof that the ultra-violet hydrogen series has been observed in the *Orion* nebula as far as Hκ, and that the line λ 3835 is due to hydrogen. But, however that may be, evidence is not lacking that the line does not belong to a series including λλ 4363, 3968, and 3869. The line in question does not appear to vary in intensity in a regular manner with the latter lines, unless indeed the relation be an inverse one. The line λ 3835 and those in Table II. to the violet of it have been observed only in the *Orion* nebula, an object in which the hydrogen lines are unusually bright, and the three lines mentioned comparatively faint. It is in the planetary nebulæ, in which the line λ 3835 is too faint for observation, that the three lines taken by Mr. Love as the basis of his series are bright.

In view of the above facts the writer considers that there is not a single coincidence between computed and observed wavelengths upon which the proposed formula can be considered to rest.

The writer considers it very doubtful whether λ 4363 is emitted under the same conditions as λλ 3968 and 3869. The latter lines are of about the same intensity in a number of different nebulæ,* but the intensity of λ 4363 appears to vary relatively to them more than can be accounted for by instrumental considerations. So far as the three corresponding *Nova* bands being characterised by an identity of structure is concerned, the observations made at the Lick Observatory show the structure of all the brighter lines to be pretty much the same.

Lick Observatory: 1902 *August.*

Explanation of Use of Tables of $\frac{1}{2}(\theta+\cos\theta)$.

By W. Steadman Aldis, M.A.

The equations connecting the eccentric, true, and mean anomalies, and radius vector in an elliptic orbit are

$$(1)\qquad m = u - e\sin u\,;$$

$$(2)\qquad \tan\frac{v}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{u}{2}\,;$$

$$(3)\qquad r = a(1 - e\cos u).$$

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* Lick Observatory Bulletin, No. 19.

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Putting $e = \sin\phi$, (1) becomes

$$\begin{aligned} m &= u - \sin\phi \sin u \\ &= \tfrac{1}{2}\{\phi + u + \cos(\phi + u)\} - \tfrac{1}{2}\{\phi - u + \cos(\phi - u)\} \\ &= C(\phi + u) - C(\phi - u) \end{aligned}$$

if $C(\theta)$ denote $\frac{1}{2}(\theta + \cos\theta)$.

The accompanying tables give the value of $C(\theta)$ in seconds for every minute of angle between $-90°$ and $+90°$.

For values beyond these limits it is easily shown that

$$\begin{aligned} C(180° - \theta) &= N - C(\theta) \\ C(-180° + \theta) &= -N - C(-\theta) \\ C(180° + \theta) &= N - C(-\theta) \end{aligned}$$

where $N = 324000$ = number of seconds in a right angle.

Examples of Use.

The Observatory, 1895 March, gave elements of Barnard's Comet.

Epoch 1895, Feb. 9·0, Berlin Mean Time.

$$M = 339° \ 8', \ \phi = 35° \ 43', \ \mu = 656'', \ \log a = 0{\cdot}4886.$$

These give the values of the mean anomaly at midnight on the dates named according to the subjoined table :

Date.	Mean Anomaly.	Date.	Mean Anomaly.
	''		''
May 26	−5584	May 31	−2304
27	−4928	June 1	−1648
28	−4272	2	− 992
29	−3616	3	− 336
30	−2960	4	+ 320

The negative sign merely indicates the anomaly as measured on the preceding side of the apse line.

To find the eccentric anomaly corresponding to mean anomaly 320'' at June 4 midnight we have

$$320 = C(\phi + u) - C(\phi - u) \quad \ldots \quad \ldots \quad (a)$$

where $\phi \qquad = 35° \ 43'$

The tables show that a change of 1' up or down in θ from 35° 43' gives a change of about 12''·48 in $C(\theta)$.

Hence the value of u to satisfy (a) will be about $\dfrac{320}{2 \times 12{\cdot}48}$ minutes; that is to say, u lies between 12' and 13'.

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Take $u = 12'$; from the tables.

$$\begin{array}{ll} C(\phi+u) = C(35^\circ\ 55') = 148173''\cdot9463 & \text{diff. for } 1' \text{ in } u = 12''\cdot3982 \\ C(\phi-u) = C(35^\circ\ 31') = 147874''\cdot2652 & \left\{ \begin{array}{l} \text{,,} \quad \text{,,} \quad \text{in } u \\ \text{downwards} = 12''\cdot5754 \end{array} \right. \\ \text{difference} = \quad 299''\cdot6811 & \end{array}$$

This is too small by $20''\cdot3189$, and as an increase of $1'$ in u would increase this on the whole by $24\cdot9736$, the required increase must be $\frac{20\cdot3189}{24\cdot9736} \times 60'' = 48''\cdot8$.

Thus $u = 12'\ 48''\cdot8$.

Take midnight, May 26; neglecting the sign,

$$m = 5584'' = C(\phi+u) - C(\phi-u).$$

A first approximation, by difference opposite $35^\circ\ 43'$, gives

$$u = \frac{5584}{2 \times 12\cdot48} \text{ minutes} = 3^\circ\ 42' \text{ about.}$$

With this value

$$\begin{array}{ll} C(\phi+u) = C(39^\circ\ 25') = 150624''\cdot8251, & \text{diff. for } 60'' = 10''\cdot9480 \\ C(\phi-u) = C(32^\circ\ \ 1') = 145075''\cdot3368 & \left\{ \begin{array}{l} \text{,,} \quad \text{,,} \\ \text{downwards} = 14''\cdot0987 \end{array} \right. \\ \text{difference} = \quad 5549''\cdot4883 & \end{array}$$

This is too small by $34''\cdot5117$, and u therefore must be increased by $\frac{34\cdot5117}{25\cdot0467} \times 60'' = 1'\ 22''\cdot7$.

Thus $u = 3^\circ\ 43'\ 22''\cdot7$.

When u is found (2) will give v. Equation (3) can be modified thus:

$$r = a(1 - e\cos u)$$

or

$$\begin{aligned} r &= a(1 - \sin\phi\cos u) \\ &= a\left\{\frac{1-\sin\overline{\phi+u}}{2} + \frac{1-\sin\overline{\phi-u}}{2}\right\} \\ &= a\left\{\left(\frac{dC}{d\theta}\right)_{\theta=\phi+u} + \left(\frac{dC}{d\theta}\right)_{\theta=\phi-u}\right\} \end{aligned}$$

Now the difference between two consecutive values of $C(\theta)$ in the tables, divided by 60, is clearly a very close approximation to the value of $\left(\frac{dC}{d\theta}\right)$ for a value of θ halfway between the two consecutive arguments.

In the last example

$$\begin{array}{rl} \phi = & 35^\circ\ 43', \\ u = & 3^\circ\ 43'\ 22''\cdot7 \end{array}$$

$$\phi+u = 39^\circ\ 26'\ 22''\cdot7, \quad \phi-u = 31^\circ\ 59'\ 37''\cdot3$$

From the tables $\left(\frac{dC}{d\theta}\right)_{\theta=39^\circ\ 26'\ 30''} = \frac{10\cdot9412}{60} = \cdot18235$

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while the value for 39° 25′ 30″ is ·18247. Thus the value of

$$\frac{dC}{d\theta} \text{ for } \theta = 39^\circ\ 26'\ 22''{\cdot}7 \text{ is } {\cdot}18236 \text{ very nearly.}$$

Similarly for

$$\theta = 31^\circ\ 59'\ 37''{\cdot}3,\ \frac{dC}{d\theta} = {\cdot}23509$$

Thus $r = a \times {\cdot}20878$

which is accurate within the limits of the data.

Taking as an example an orbit of large eccentricity and also taking a large mean anomaly, that of Halley's Comet, Epoch 1910 (from *Encyclopædia Britannica*)

$a = 17{\cdot}9555$, Period 76·085 yrs., $e = {\cdot}961733$, T = May 26·83.

Hence on May 26·83, 1900, the mean anomaly of the comet was

$$\frac{10 \times 360 \times 60 \times 60}{76{\cdot}085} \text{ seconds} = 170335''{\cdot}8 \text{ nearly}$$

$$= 31^\circ\ 48'\ 55''{\cdot}8 \text{ nearly}$$

Since $m = u - e \sin u$

$u = m + e \sin m.$ 1st approximation ... (A)

$= m + (e + e^2 \cos m) \sin m$

$= m + e' \sin m.$ 2nd approximation ... (B)

With the 1st approximation

$$u = \mathrm{C}(m - \phi) - \mathrm{C}(-m - \phi)$$

But $m =$ 31° 48′ 55″·8

ϕ 74 5 53·8

$m - \phi = -42$ 16 58 $= -$ 42° 17′ approximately

$m + \phi$ 105 55 „

Now C (−105° 55′) = −N − C (−74° 5′).

Hence the first approximation to the value of u is given by

$\mathrm{C}\ (m - \phi) =$ 190″·1194

N = 324000·

324190·1194

C (−74° 5′) = −105067·0764

and $u =$ 219123·0430

= 60° 52′ 3″ nearly.

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Take as a trial value $u = 60^\circ\ 52'$

$$\phi = 74\ \ 6$$

$$C\ (\phi+u) = C\ (180^\circ-45^\circ\ 2') = N-C\ (45^\circ\ 2')$$

$$C\ (\phi-u) = C\ (13^\circ\ 14')$$

$$\begin{array}{rll}
C\ (\overset{\circ}{45}\ \ \overset{\prime}{2}) = & 153943''\cdot1829 & \text{diff. for } 60'' = \ \ 8''\cdot7713 \\
C\ (13\ \ 14) = & 124213\cdot8138 & \ \ ,,\qquad ,,\qquad 23\cdot1283 \\
\text{sum} = & 278156\cdot9967 & \\
 & \underline{324000\cdot} & \\
m = & 45843\cdot0033 &
\end{array}$$

The deficiency in the value of m is $134492''\cdot8$. The increase in the value of m for a change of $1'$ in u is at the present point about $8''\cdot8+23''$, or roughly $32''$. If this continued uniform, an addition of $\frac{134492\cdot8}{32}$ minutes would have to be made to u, or of $70^\circ\ 31'$.

This would make

$$u = 130^\circ\ 55',\quad \phi+u = 205^\circ\ 1' = 180^\circ+25^\circ\cdot1$$

$$\phi-u \qquad\qquad = -56^\circ\ 49'$$

and at this point a change of $1'$ in u would make a total change of $42''\cdot7+55''\cdot1$, or nearly $98''$, in m; nearly three times the amount at starting. If we assume an average change of double the original value of the increment, the required change in u will be 35°. This gives $u = 95^\circ\ 52'$

$$\therefore\ (\phi+u) = 169^\circ\ 58' = 180-10^\circ\ 2'$$

$$(\phi-u) = -21^\circ\ 46'.$$

Hence $m = N-C\ (10^\circ\ 2')-C\ (-21^\circ\ 46')$.

Now

$$\begin{array}{rll}
C\ (\overset{\circ}{10}\ \ \overset{\prime}{2}) = & 119615''\cdot1541, & \text{diff. for } 1' = 24''\cdot7691 \\
C\ (-21\ \ 46) = & \underline{\ 56599\cdot2405} & \ \ ,,\qquad ,,\qquad 41\cdot1289 \\
 & 176214\cdot3946 & \\
 & \underline{324000\cdot} & \\
\text{Thus}\qquad m = & 147785\cdot6054 &
\end{array}$$

The deficiency is now $22550''$, and the change in m for an increase of $60''$ in u is about $66''$. This change increases as u increases, and if it be assumed on an average to be about $70''$, an increment to u of about 5° is needed.

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Thus $u = 100^\circ\ 52'$

and $m = \mathrm{N} - \mathrm{C}\ (5^\circ\ 2') - \mathrm{C}\ (-26^\circ\ 46')$

$$\begin{array}{rrl}
\mathrm{C}\ (5^\circ\ 2') = & 111794''\cdot 7, & \text{diff. for } 1' = 27''\cdot 4 \\
\mathrm{C}\ (-26\ 46) = & \underline{43901\cdot 6} & \quad ,, \qquad ,, \qquad 43\cdot 5 \\
 & 155696\cdot 3 & \\
 & \underline{324000} & \\
m = & 168303\cdot 7 &
\end{array}$$

deficiency now $= 2032''\cdot 1$, and as the change in m for a change of $60''$ in u is about $71''$, this requires an addition to u of about $\frac{2032\cdot 1'}{71}$, or between $28'$ and $29'$.

Taking $28'$, $u = 101^\circ\ 20'$, and repeating the process, if it is desired to be very accurate, taking $\phi = 74^\circ\ 5'\ 53''\cdot 8$ instead of $74^\circ\ 6'$ it will turn out that the correct value is $101^\circ\ 20'\ 33''\cdot 2$, the error being less than a tenth of second.

The first guess can be often made more correct by taking (B) instead of (A), and finding an angle ϕ' such that

$$\sin \phi' = e + e^2 \cos m, \text{ whence (B) gives}$$

$$u = \mathrm{C}\ (m - \phi') - \mathrm{C}\ (-m - \phi')$$

as a first approximation. Or if $e + e^2 \cos m$ is greater than unity, writing 90° for ϕ'

$$u = \mathrm{C}\ (m - 90^\circ) - \mathrm{C}\ (-m - 90^\circ).$$

When u is known, r can be found from the column of differences, as in the former example.

Part 2.

A second use to which the tables can be applied is that of finding the elements of an elliptic orbit when two focal distances, the angle between them, and the time occupied in passing from one position to the other are known.

If (r, θ), (r', θ') be the focal polar coordinates of the two positions, and t the time occupied in passing between them, it is known that the following equation holds :

$$\frac{kt}{a^{\frac{3}{2}}} = (\epsilon - \sin \epsilon) - (\delta - \sin \delta) \quad \ldots \quad \ldots \quad (1)$$

where

$$\cos \delta = 1 - \frac{r + r' - c}{2a} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

$$\cos \epsilon = 1 - \frac{r + r' + c}{2a} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

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c being the length of the chord joining the two points and k the measure of the attractive force at unit distance.

Equation (1) can be written

$$\frac{kt}{a^{\frac{3}{2}}}=\left\{\frac{\pi}{2}-\delta+\cos\left(\frac{\pi}{2}-\delta\right)\right\}-\left\{\frac{\pi}{2}-\varepsilon+\cos\left(\frac{\pi}{2}-\varepsilon\right)\right\}$$

or
$$\frac{\frac{1}{2}kt}{a^{\frac{3}{2}}}=\mathrm{C}(90-\delta)-\mathrm{C}(90-\varepsilon)\quad\ldots\quad\ldots\quad\ldots\quad\ldots\quad(4)$$

if k be expressed in seconds and δ and ε in degrees and parts of a degree.

The angle δ lies between 0° and 180°, or between 180° and 360°, according as $\theta\sim\theta'$ is less or greater than 180°, while ε is only restricted to lie between 0° and 360° (Gauss, *Theoria Motus*, ed. 2, Gotha, 1877, p. 132).

The three equations (1), (2), (3) are sufficient to determine δ, ε, and a. The last two can be written

$$\sin^2\tfrac{1}{2}\delta=\frac{r+r'-c}{4a}\quad\ldots\quad\ldots\quad\ldots\quad(5)$$

$$\sin^2\tfrac{1}{2}\varepsilon=\frac{r+r'+c}{4a}\quad\ldots\quad\ldots\quad\ldots\quad(6)$$

whence
$$\sin\tfrac{1}{2}\delta=\sqrt{\frac{r+r'-c}{r+r'+c}}\,.\,\sin\tfrac{1}{2}\varepsilon\quad\ldots\quad\ldots\quad(7)$$

The following process of successive approximation will give the value of a to as great a degree of accuracy as the data allow.

Assume any value for ε. From (7) the corresponding value of δ can be deduced, or, as is more useful in practice, a can be first derived from (6) and then δ from (5).

Placing the values of δ and ε in (4), by means of the tables of $\mathrm{C}(\theta)$ another value of a is obtained. Let the values of a given by (6) and (4) be called x and y respectively.

Repeat this process with another assumed value of ε, and let x' and y' be the corresponding quantities on the second assumption. A comparison of $x'\sim y'$ with $x\sim y$ will indicate which of the two assumed values of ε is nearer the truth, and, with practice, will also give to a considerable degree of correctness the distance of the required value from the original assumption.

Taking an example worked out by Gauss (*Theoria Motus*, p. 103), we have

$$\log r=0{\cdot}3307640$$
$$\log r'=0{\cdot}3222239$$
$$\theta'-\theta=7^{\circ}\ 34'\ 53''{\cdot}73$$
$$t=21{\cdot}93391\text{ days}$$

Also
$$\log k=3{\cdot}55000\,65746$$
$$\log t=1{\cdot}34111\,60571$$

whence
$$\log(\tfrac{1}{2}kt)=4{\cdot}59009\,26360$$

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Ordinary trigonometrical processes give

$$c = 0\cdot2835079$$

while $$r+r' = 4\cdot2417487$$

whence $$r+r'+c = 4\cdot5252566$$

... ... $$r+r'-c = 3\cdot9582408$$

And $$\log\frac{r+r'+c}{4} = 0\cdot0535832$$

$$\log\frac{r+r'-c}{4} = 9\cdot9954422$$

For a first guess it is convenient to take $90^\circ-\epsilon = 0$.

Thus $$\mathrm{L}\sin\tfrac{1}{2}\epsilon = 9\cdot8494850$$

$$\log\sin^2\tfrac{1}{2}\epsilon = 9\cdot6989700$$

... ... $$\log\frac{r+r'+c}{4} = 0\cdot0535832$$

Hence $$\log x = 0\cdot3546132$$

... ... $$\log\frac{r+r'-c}{4} = 9\cdot9954422$$

By (5) $$\log\sin^2\tfrac{1}{2}\delta = 9\cdot6408290$$

$$\mathrm{L}\sin\tfrac{1}{2}\delta = 9\cdot8204145$$

Whence $\tfrac{1}{2}\delta = 41^\circ\ 24'\ 3''\cdot4$ and $\delta=82^\circ\ 48'\ 6''\cdot8$

$$(90-\delta) = 7^\circ\ 11'\ 53''\cdot2$$

From the tables $\mathrm{C}\ (7^\circ\ 11') = 115252''\cdot9290$

diff. for $60'' = 26''\cdot2443$

Whence, neglecting second differences at this stage,

$$\mathrm{C}\ (7^\circ\ 11'\ 53''\cdot2) = 115276''\cdot1989$$

Also $$\mathrm{C}\ (90-\epsilon) = 103132''\cdot4031$$

$$\text{difference} = 12143''\cdot7958$$

The logarithm of this $= 4\cdot0843545$

$$\log\tfrac{1}{2}(kt) = 4\cdot5900926$$

$$\log a^{\frac{3}{2}} = \text{the difference} = 0\cdot5057381$$

$$\begin{array}{r} 2 \\ \hline 3)1\cdot0114762 \\ \hline \end{array}$$

$$\log y = 0\cdot3371587$$

The values of x and y are nearer than they usually come at the first guess. Experience, or the theory of the equations (6) and (4), shows that if ϵ be diminished both x and y increase, but y increases faster than x; and if ϵ be increased x and y decrease,

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the rate of change of y being the greater. As here y is less than x, it follows that in order to produce the required equality between them both must be increased, and ϵ and δ consequently diminished.

As a second guess take $\epsilon = 80^\circ$. By the same process as above $\delta = 73^\circ\ 54'\ 26''{\cdot}73$.

$$\log x = 0{\cdot}4374482$$
$$\log y = 0{\cdot}4407142$$

Thus y is now greater than x, and the second value of ϵ is too small.

Roughly, the values of x and y for $\epsilon = 90^\circ$ and $\epsilon = 80^\circ$ are $x_1 = 2{\cdot}2626$, $y_1 = 2{\cdot}1735$, $x_2 = 2{\cdot}7381$, $y_2 = 2{\cdot}7588$. Thus

$$x_1 - y_1 = {\cdot}0891,\ y_2 - x_2 = {\cdot}0207$$

and, assuming that $x \backsim y$ changes uniformly with ϵ, this would give as the correct value of ϵ about $81^\circ\ 53'\ 7''$.

In the later stages of approximation it is generally best to take two trial values of ϵ between which the real value lies, the simultaneous working of which is not much more labour than that of one separately, as far as reference to tables is concerned. A little practice will show what can easily be proved, that the rate of change of $y \backsim x$ increases as δ decreases, and *vice versa.* Hence the real value of ϵ in this case is less than $81^\circ\ 53'\ 7''$. Taking as two trial values for ϵ, $81^\circ\ 54'$ and $81^\circ\ 30'$, the corresponding values of δ are $75^\circ\ 36'\ 30''{\cdot}6$ and $75^\circ\ 15'\ 2''{\cdot}74$, while the values of log x and log y are given by

$$\left.\begin{aligned}\log x_1 &= 0{\cdot}4205700\\ \log y_1 &= 0{\cdot}4201511\end{aligned}\right\}\ \text{and}\ \ \begin{aligned}\log x_2 &= 0{\cdot}4240764\\ \log y_2 &= 0{\cdot}4244420\end{aligned}$$

The required value of ϵ must evidently therefore lie between $81^\circ\ 30'$ and $81^\circ\ 54'$, and not very far from halfway between them. As another pair of trial values taking $\epsilon = 81^\circ\ 40'$ and $\epsilon = 81^\circ\ 42'$, by the same process it will be found that

$$\left.\begin{aligned}\log x_1 &= 0{\cdot}4226124\\ \log y_1 &= 0{\cdot}4226428\end{aligned}\right\}\ \ \ \begin{aligned}\log x_2 &= 0{\cdot}4223202\\ \log y_2 &= 0{\cdot}4222933\end{aligned}$$

The real value of ϵ is thus intermediate between these two.

The value of a must therefore lie between x_1 and x_2: that is, between 2·6461 and 2·6443. In many cases this will be near enough, but if greater accuracy is required a pair of closer values may be tried, from which it will be found that for $\epsilon = 81^\circ\ 41'\ 11''{\cdot}5$, the values of log x and log y coincide for seven figures, each of them being 0·4224385. The value given by Gauss is 0·4224389, the discrepancy being within the limits of error accumulating in the use of seven-figure logarithms.

The discovery of the value of a, by means of the equations

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(4), (6), and (7), may be considered as the discovery of the point of intersection of two curves whose equations are of the forms

$$y = \frac{\lambda}{\sin^2 \frac{1}{2}x} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

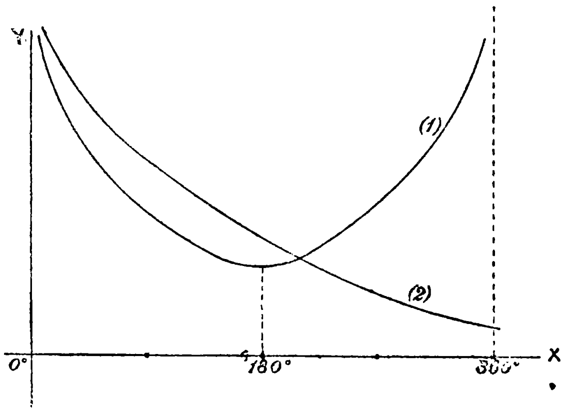
$$\frac{\mu}{y^{\frac{3}{2}}} = (x' - \sin x') - (x - \sin x) \quad \ldots \quad \ldots \quad (2)$$

where x' is a function of x given by an equation of the form

$$\sin \tfrac{1}{2}x' = \nu \sin \tfrac{1}{2}x,$$

λ, μ, ν being given constants.

The ordinate of the point of intersection is the required value of a.



The general form of these curves is as represented in the diagram. The axis of y is asymptotic to both, but, while the ordinate of (1) decreases until $x = 180°$, and then increases till it becomes infinite again when $x = 360°$, the ordinate of (2) continually decreases as x passes from 0° to 360°. It is not difficult to prove that in all cases of elliptic orbits the ordinate of (2) is greater than that of (1) for very small values of x. Obviously the two curves will cut each other in one point, and one point only, between the permissible limits of the value of x.

If then the values of the ordinate for each of the curves be calculated for two assumed values of x, as in the example worked out, and the points be marked down on a piece of scale paper, curves of the required general shape can be roughly drawn through the pairs of points thus found, and the point of their intersection will give an approximate starting point for the closer calculation of the correct value.

When in any example the value of a has been thus obtained, the values of the eccentricity and semilatus rectum can be found.

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The formulæ for this purpose are easily shown to be

$$p = \frac{2rr'\sin^2\Delta}{c}\tan\tfrac{1}{2}\psi \text{ or } \frac{2rr'\sin^2\Delta}{c}\cot\tfrac{1}{2}\psi$$

where p is the semilatus rectum, 2Δ the angle between the given radii vectores, and ψ an acute angle determined by the equation

$$\left(r+r'-\frac{rr'\cos^2\Delta}{a}\right)\sin\psi = c.$$

In the formula for p the value $\tan\frac{1}{2}\psi$ or $\cot\frac{1}{2}\psi$ has to be taken according as 2Δ, or $\theta'-\theta$ is greater or less than 180°. When p is known and a, the value of e can be obtained from the formula $p = a(1-e^2)$, or if $e = \sin\phi$, $\cos\phi = \sqrt{\frac{p}{a}}$.

In using this method for finding a it is better to make an assumption for the value of ϵ than for that of δ although the range of possible values for the former is double that for the latter, for two reasons. First, for any assumed value of ϵ, equation (7) only gives one value of δ, while for an assumed value of δ it would give two possible ones for ϵ; secondly, any assumed value for ϵ will give a real value for δ, while this would not always be the case if the opposite method were adopted.

In the tables negative values of $C(\theta)$ are denoted by a black line at the right of the column.

In cases where there was a little doubt as to whether the figure after the fourth decimal ought to be 5+ or 5−, so that there was uncertainty as to the exact value of the last figure retained, that value has been chosen which made the sequence of the second differences most regular.

A dot placed after a decimal whose last significant figure is 5, thus ·6345· or ·6350· denotes that the real value is slightly less. This enables the last digit or two digits to be cut off with accuracy in all cases.

The writer desires to acknowledge his obligation to Professor G. H. Darwin for valuable hints as to the construction and the method of application of the Tables.

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Δ².	Diff.	0°.	′	−0°.	Diff.	Δ².	Δ².	Diff.	1°.
	″	″		″	″			″	″
86	29·9957	103132·4031	0	103132·4031	30·0043	86	87	29·4721	104916·6956
88	·9869	162·3988	1	102·3988	·0131	88	88	·4633	946·1677
87	·9782	192·3857	2	072·3857	·0218	87	87	·4546	104975·6310
88	·9694	222·3639	3	042·3639	·0306	88	87	·4459	105005·0856
87	·9607	252·3333	4	103012·3333	·0393	87	88	·4371	034·5315.
87	·9520	282·2940	5	102982·2940	·0480	87	86	·4285	063·9686
87	·9433	312·2460	6	952·2460	·0567	87	88	·4197	093·3971
87	·9346	342·1893	7	922·1893	·0654	87	87	·4110	122·8168
88	·9258	372·1239	8	892·1239	·0742	88	88	·4022	152·2278
87	·9171	402·0497	9	862·0497	·0829	87	86	·3936	181·6300
87	·9084	431·9668	10	831·9668	·0916	87	88	·3848	211·0236
88	·8996	461·8752	1	801·8752	·1004	88	87	·3761	240·4084
87	·8909	491·7748	2	771·7748	·1091	87	88	·3673	269·7845.
87	·8822	521·6657	3	741·6657	·1178	87	86	·3587	299·1518
87	·8735	551·5479	4	711·5479	·1265	87	88	·3499	328·5105.
88	·8647	581·4214	15	681·4214	·1353	88	87	·3412	357·8604
87	·8560	611·2861	6	651·2861	·1440	87	88	·3324	387·2016
87	·8473	641·1421	7	621·1421	·1527	87	86	·3238	416·5340
87	·8386	670·9894	8	590·9894	·1614	87	88	·3150	445·8578
88	·8298	700·8280	9	560·8280	·1702	88	87	·3063	475·1728
87	·8211	730·6578	20	530·6578	·1789	87	87	·2976	504·4791
87	·8124	760·4789	1	500·4789	·1876	87	88	·2888	533·7767
87	·8037	790·2913	2	470·2913	·1963	87	87	·2801	563·0655
88	·7949	820·0950.	3	440·0950.	·2051	88	87	·2714	592·3456
87	·7862	849·8899	4	409·8899	·2138	87	87	·2627	621·6170
87	·7775	879·6761	25	379·6761	·2225	87	87	·2540	650·8797
88	·7687	909·4536	6	349·4536	·2313	88	88	·2452	680·1337
87	·7600	939·2223	7	319·2223	·2400	87	87	·2365	709·3789
87	·7513	968·9823	8	288·9823	·2487	87	88	·2277	738·6154
87	·7426	103998·7336	9	258·7336	·2574	87	86	·2191	767·8431
88	·7338	104028·4762	30	228·4762	·2662	88	88	·2103	797·0622
87	·7251	058·2100	1	198·2100	·2749	87	87	·2016	826·2725
87	·7164	087·9351	2	167·9351	·2836	87	87	·1929	855·4741
88	·7076	117·6515.	3	137·6515.	·2924	88	87	·1842	884·6670
86	·6990	147·3591	4	107·3591	·3010	86	88	·1754	913·8512
88	·6902	177·0581	35	077·0581	·3098	88	87	·1667	943·0266
87	·6815	206·7483	6	046·7483	·3185	87	87	·1580	105972·1933
87	·6728	236·4298	7	102016·4298	·3272	87	87	·1493	106001·3513
88	·6640	266·1026	8	101986·1026	·3360	88	88	·1405	030·5006
87	·6553	295·7666	9	955·7666	·3447	87	86	·1319	059·6411
87	·6466	325·4219	40	925·4219	·3534	87	88	·1231	088·7730
88	·6378	355·0685.	1	895·0685.	·3622	88	88	·1143	117·8961
87	·6291	384·7063	2	864·7063	·3709	87	86	·1057	147·0104
87	·6204	414·3354	3	834·3354	·3796	87	88	·0969	176·1161
87	·6117	443·9558	4	803·9558	·3883	87	87	·0882	205·2130
87	·6030	473·5675	45	773·5675	·3970	87	87	·0795	234·3012
88	·5942	503·1705.	6	743·1705.	·4058	88	87	·0708	263·3807
87	·5855	532·7647	7	712·7647	·4145	87	88	·0620	292·4515.
87	·5768	562·3502	8	682·3502	·4232	87	87	·0533	321·5135.
88	·5680	591·9270	9	651·9270	·4320	88	87	·0446	350·5668
87	·5593	621·4950	50	621·4950	·4407	87	87	·0359	379·6114
87	·5506	651·0543	1	591·0543	·4494	87	88	·0271	408·6473
87	·5419	680·6049	2	560·6049	·4581	87	87	·0184	437·6744
88	·5331	710·1468	3	530·1468	·4669	88	87	·0097	466·6928
87	·5244	739·6799	4	499·6799	·4756	87	87	29·0010	495·7025
87	·5157	769·2043	55	469·2043	·4843	87	87	28·9923	524·7035
87	·5070	798·7200	6	438·7200	·4930	87	87	·9836	553·6958
88	·4982	828·2270	7	408·2270	·5018	88	88	·9748	582·6794
86	·4896	857·7252	8	377·7252	·5104	86	87	·9661	611·6542
88	29·4808	887·2148	9	347·2148	30·5192	88	88	28·9573	640·6203
		104916·6956	60	101316·6956					106669·5776